Indian Statistical Institute
Second Semester Final Examination: 2022-23

Course Name: M. Math, 1^{st} year
Subject Name : Topology II
Maximum Marks: 50, Duration: 3 hours
Date: 08.05.2023

- Answer as many questions as you can.
- Maximum marks is 50.
- You may use any results proved in class. Any other results (including those in homework problem sets) require proof.
- Credit for the solutions will be given only when proper justifications are provided for them.

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1. Let $\mathbb{C}P^n \to \mathbb{C}P^{n+k}$ be the standard inclusion. Compute the homology groups of $\mathbb{C}P^{n+k}/\mathbb{C}P^n$. 9

2. Let $f_k : S^n \to S^n$ be a map of degree k, and let $\text{Cyl}(f_k)$ denote the mapping cylinder of f_k.
   a) Prove that $H_n\text{Cyl}(f_k) \cong \mathbb{Z}$. 2
   b) Construct a map $g_k : \text{Cyl}(f_k) \to S^n$ which induces multiplication by k on H_n. 3
   c) Compute the degree of the composite
   $$S^n \to \text{Cyl}(f_k) \xrightarrow{g_k} S^n,$$
   where $S^n \to \text{Cyl}(f_k)$ is the standard inclusion. 4

3. Let C_2 act on $S^n \times S^n$ by the formula $\sigma(x, y) = (-x, -y)$. Let $P = (S^n \times S^n)/C_2$.
   a) Prove that $S^n \times S^n \to P$ is a covering space. 4
   b) Prove that every map from P to S^1 is null-homotopic. 5

4. Let X be a simply connected CW complex of dimension $< n$, and $f : X \to \mathbb{R}P^n$ be a map. Show that $f_* : \tilde{H}_*(X) \to \tilde{H}_*(\mathbb{R}P^n)$ is the 0-map. 9

5. For a map $f : X \to X$, let $T_f = X \times [0, 1]/((x, 0) \sim (f(x), 1))$ denote the mapping torus. For a reflection $\rho : S^2 \to S^2$, calculate the homology of T_ρ. 9

6. Let $f : S^m \to S^n$ be a map, with $m > n$. Let $Y = C(f)$, the mapping cone of f.
   a) Compute the homology groups of Y. 3
   b) For a CW complex X, prove that $H_k(X \times Y) \cong H_k(X) \oplus H_{k-n}(X) \oplus H_{k-m-1}(X)$. 6