Assignment Set I

Unless otherwise specified, any surface will mean a level set for some regular maps as in class. Vector fields will mean locally defined smooth vector fields.

Qn 1. Let X, Y be two vector fields on an m-surface surface $S \subset \mathbb{R}^n$ $(m < n)$ and let $\tilde{X}, \tilde{Y}$ be extensions of X, Y respectively to vector fields on $\mathbb{R}^n$. Prove that $[\tilde{X}, \tilde{Y}](p) = [X, Y](p)$ for all points $p \in S$.

Qn 2. Prove that two vector fields X and Y commute (i.e. $[X, Y] = 0$) if and only if the corresponding local one parameter groups τ_t^X and τ_s^Y commute for all values of s, t for which they are defined.

Besides, solve exercise no. **12.8, 12.9 and 12.13 from Thorpe's book , page 93, Chapter 12.**

Note : For 12.13, first describe the graph as a level set in $\mathbb{R}^{n+1}$. You may also need to compute the determinant of a matrix of the form $\begin{pmatrix} I & v \\ v' & 0 \end{pmatrix}$, where $v \in \mathbb{R}^n$ as a column vector and v' its transpose row vector and I is the $n \times n$ identity matrix. You may use induction to prove a formula for the determinant of such a matrix.